Minh Nguyen

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4.1 Weak Induction

5.1 (7th edition): { 6,8,14,18,28,38,40}

6) let P(n) be the sum of the the n terms is correct.

Basis Step: P(1) is true, 1 \* 1! - (1 + 1)! - 1 -1

Inductive Step: The statement that P(k) is true, where k is a positive integer is the inductive hypothesis.

So P(k) is 1 \* 1! + 2 \* 2! + ... + k \* k! - (k + 1)!

To complete this, we must show that P(k+1) is also true if P(k) is true. Add (k +1) \* (k+1)! to both sides.

(k+1)! 1+ (k+1) (k+1)! = (k + 1)! \* 1 + (k + 1)! \*(k + 1) - 1

= (k+1)![k+2] - 1

=(k + 2)! - 1

This completes the inductive argument showing P(k+1) is true.

8)Solution: Let P(n) be the sum of the first n terms

Basis Step: P(0) is true because 2 = = 2

Inductive Step: To prove that the inductive step is true, if P(k) is true, then P(k+1) is also true.

P(k) is in this case: 2 - 2\*7 + 2\* 72 - .... + 2(-7)k = (1 - (-7)k+1) / 4

Adding 2(-7)k+1 to both sides of the equality of P(k):

2 - 2\*7 + 2\* 72 - .... + 2(-7)k + 2(-7)k+1 = (1 - (-7)k+1) / 4 + 2(-7)k+1

= +

= +

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This shows P(k+1) is also true when P(k) is true, thus completing the inductive step

14) Solution: Let P(n) be the sum of the n terms

Basis step: P(1) is true, because 1(2)1 = (1-1)21+1 +2

2 = 2

Inductive step: To prove the inductive argument, the inductive hypothesis is that P(k) is true, where k is a postive integer. P(k) is the statement

If P(k) is true, then we must show that P(k+1) is also true. So we add (k+1)\* 2k+1 to both sides of the equation of P(k):

= 2k+1 \* (k - 1)+2k+1 \* (k+1) + 2

= 2k+1 [k -1 + k + 1] + 2

= 2k+1 \* 2k + 2

= k \* 2k+2 + 2

=

Here we finish the inductive argument, and it is true for P(k+1) so our argument is complete.

18) a) P(2) = (2)! < 22 ::::: 2 < 4

b) 2 is less than 4

c) Inductive hypothesis P(k) is k! < kk

d) For all k > 2 that P(k) means P(k+1) is true; so show that (k+1) < (k+1)k+1

e) k! \* (k+1) < kk  (k+1) < (k+1)k(k+1) - (k+1)k+1

f) We've completed the basis step and the inductive step, proving that P(k) and P(k+1) is true, meaning that we've completed the proof through mathematical induction.

28) Prove n2 - 7n + 12 is nonnegative whenever n >= 3

Solution: Let P(n) be the statement that n2 - 7n + 12 is nonnegative whenever n >= 3

Basis Step: P(3) = 32 - 7(3) + 12 = 9 - 21 + 12 = 0 . 0 is a nonnegative number. Basis step complete

Inductive Step: The inductive hypothesis P(k) is true, where k is a positive integer greater than 3. P(k) is

k2 - 7k +12 >= 0 (k >= 3)

If P(k) is true, then we must show P(k+1) is also true to complete the inductive step.

(k+1)2 - 7(k+1) + 12 = k2 + 2k + 2 - 7k - 7 + 12

= (k2 - 7k + 12) + 2(k-3)

Since whenever k>=3, we have a nonnegative number in 2(k-3), and the other portion of the equation is the same as the original equation, then we know that adding to it will always lead to a nonnegative number, thus proving that P(k)is true, and makes P(k+1) also true.

Completing the basis step and the induction step, we have prove that P(n) is true for all n>= 3.

38) Solution: Let P(n) be U Aj ⊆ UBj if Ai ⊆ Bi for i -1,2,...,n

Basis step: P(1) is proof that A1 ⊆ B1 , so it implies that U Aj ⊆ UBj

Inductive Step: The inductive hypothesis is the statement that P(k) is true, where k is a positive integer. P(k) is then the statement if Aj ⊆ Bj for j -1,2,...,k ,then U Aj ⊆ UBj

if x is an element in the above subsets, then x ∈ UAj or x ∈ Ak+1. if x ∈ U, so is x ∈ UBj

But if x ∈ Ak+1, then we also know that Ak+1 ⊆ B+k+1 , which also means x ∈ Bk+1. So with both possibility leading to the fact that x is an element of both cases for B, it shows that P(k+1) is true, which completes our inductive case.

40) Solution: Let P(n) be (A1 ∩ A2 ∩ ... ∩ An) U B = (A1 U B) ∩ (A2 ∩ B) ∩ ... ∩ ( An U B)

Basis step: P(1) is true because A1 U B = A1 U B

Inductive step: The inductive hypothesis is the statement that P(k) is true, where k is a positive integer. The statement P(k) is (A1 ∩ A2 ∩ ... ∩ Ak) U B = (A1 U B) ∩ (A2 ∩ B) ∩ ... ∩ ( Ak U B)

If P(k) is true, we must show that P(k+1) is also true.

(A1 ∩ A2 ∩ ... ∩ Ak ∩ Ak+1 ) U B = [(A1 ∩ A2 ∩ ... ∩ Ak ∩ Ak+1)] U B - by associative law

= [(A1 ∩ A2 ∩ ... ∩ Ak) U B ] ∩ (Ak+1 U B) - distributive law

= (A1 U B) ∩ (A2 ∩ B) ∩ ... ∩ ( Ak U B) ∩ (Ak+1 U B) by the IH

So, P(k+1) is true. Completing our inductive argument.